

IMM Advanced Probability Fall 2025

Part III Outline Dec 7-18

Basic references:

- [1] D. Williams, Probability with Martingales
- [2] R. Durrett, Probability: Theory and Examples
- [3] own lecture notes

Lectures:

There will be 24 hours of teaching divided into 8 lectures as follows.

L1 (Dec 7): Review of martingales. The Kolmogorov strong law of large numbers (SLLN) proven using martingales and the more general SLLN for martingales.

L2 (Dec 8): Concentration of measure inequalities (i.e. large deviations type estimates): Hoeffding's inequality. Azuma-Hoeffding inequality for martingales.

L3 (Dec 10): Basic properties of characteristic functions (chapter 16 in [1]). Weak convergence of probability measures (chapter 17 in [1]).

L4 (Dec 11): The Central Limit Theorem (Statement and proof/examples). Other applications of characteristic functions (chapter 18 in [1]). The Lindeberg-Feller theorem. Berry-Esseen theorem (just the statement).

→ Hand-in exercise list.

L5 (Dec 14): Poisson convergence, namely the law of rare events, examples, Poisson processes (section 3.6 in [2]). Definition of Brownian motion (section 8.1 in [2]).

L6 (Dec 15): Construction of Brownian motion. Stopping times (sections 8.1, 8.2 in [2]).

L7 (Dec 17): Extension of the CLT (by means of Brownian motion), namely Donsker's theorem (section 8.7 in [2]).

L8 (Dec 18): Either one of the following two topics:

- (i) Ergodic theory: definitions, examples, Birkhoff's ergodic theorem
- (ii) Markov chains: definition, examples, the Markov property, the transition operator, stationary measure, mixing.

→ Hand-in exercise list.